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# TWO DIMENSIONAL QCD AND ABELIAN BOSONIZATION

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## ABSTRACT

A bosonized action, that reproduces the structure of the 't Hooft equation for $QCD_2$ in the large-$N$ limit, up to regularization dependent terms, is derived.

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The aim of this paper is to show that the effective action for mesons in $QCD_2$ is a local one and can be computed by means of abelian bosonization. As a partial check I also show that the structure of the various terms that appear in the 't Hooft equation [1] is reproduced, up to regularization dependent terms, when quantum fluctuations are computed for this action in the large-$N$ limit. I leave for a forthcoming paper the check that also the coefficients of each term agree with the 't Hooft equation .
To be precise, in this paper I consider a slightly more general version of $QCD_2$ which is referred to as the generalized 't Hooft model and has been introduced in the literature in ref. [2]. In the case of the (generalized) Schwinger model, the authors of ref. [2] have shown how standard bosonization gives an effective action of mesons interacting with a Landau-Ginzburg potential. In the present paper I show how to get the meson effective action in the non-abelian case, starting with the Lagrangian of the generalized 't Hooft model:

$$L = \frac{N}{8\pi} Tr(E\epsilon^{\mu\nu}F_{\mu\nu}) - \frac{N}{4\pi} g^2 \sum_2^\infty f_n Tr E^n + \bar{\psi}(iD_\mu\gamma^\mu - m)\psi \tag{1}$$

This Lagrangian reduces to $QCD_2$ with one flavour for $f_2 = \frac{1}{8\pi}$ and all other $f_n$ vanishing. In the $U(1)$ case, the meson effective action follows immediately from standard bosonization of the fermionic part of the Lagrangian plus axial gauge-fixing of the gauge field [2].
In the non-abelian case, two more crucial steps are necessary for being able to perform abelian bosonization. The $U(N)$ and $SU(N)$ case can be considered at the same time. In the following formulae we can go from $U(N)$ to $SU(N)$ simply imposing the traceless condition on all the diagonal Lie-algebra generators. First, it is convenient to fix the gauge partially, reducing the $U(N)$ gauge symmetry to a diagonal $U(1)^N$, setting equal to zero the charged components of $E$:

$$E^{ch} = 0 \tag{2}$$

After doing so, the integration over the charged gluons gives the effective Lagrangian:

$$
\begin{aligned}
L &= \frac{N}{8\pi}\sum_k E_k \epsilon^{\mu\nu}(\partial_\mu A^k_\nu - \partial_\nu A^k_\mu) - \frac{N}{4\pi} g^2 \sum_2^\infty f_n \sum_k E^n_k + \\
&+ \sum_k \bar{\psi}^k [(i\partial_\mu - A^k_\mu)\gamma^\mu - m]\psi^k - \frac{4\pi i}{N}\sum_{k\neq j}\frac{\bar{\psi}^k\gamma^1\psi^j\bar{\psi}^j\gamma^0\psi^k}{E_k - E_j}
\end{aligned}
\tag{3}
$$

where the sum on the indices $k$ and $j$ now runs only over the neutral diagonal components of the $E$-field. Remarkably the effective action in Eq.(3) contains a local four fermion interaction of the charged fermionic currents.
This effective Lagrangian has been obtained integrating over all paths the exponential of $i$ times the action associated to the Minkowski Lagrangian of Eq.(1). I have choosen to perform Minkowski path integrals instead of Euclidean ones only for convenience, in order to use standard bosonization formulae. I have not included in the Minkowski effective action the Faddeev-Popov determinant, that comes from the gauge-fixing, and the determinant that comes from the integration over the charged gluon fields [3]. They will be included only at the end of the computation, after analytical continuation from the Minkowski to the Euclidean region. In any case, their contribution to the Euclidean Lagrangian (defined in such a way that the exponential of minus the Euclidean action enters the path integral) is given by:

$$
-\frac{1}{4\pi}\sum_{k>j} R\log(E_k - E_j)^2 \tag{4}
$$

where $R$ is the Riemannian curvature of the surface on which the theory lives. The second step consists in a Fierz rearranging of the four fermion term, that produces a new four fermion interaction involving neutral chiral currents:

$$
\begin{aligned}
L &= \frac{N}{8\pi}\sum_k E_k \epsilon^{\mu\nu}(\partial_\mu A^k_\nu - \partial_\nu A^k_\mu) - \frac{N}{4\pi} g^2 \sum_2^\infty f_n \sum_k E^n_k + \\
&+ \sum_k \bar{\psi}^k [(i\partial_\mu - A^k_\mu)\gamma^\mu - m]\psi^k +
\end{aligned}
$$

$$+\frac{2\pi i}{N}\sum_{k\neq j}\frac{\bar{\psi}^k(1+\gamma^5)\psi^k\bar{\psi}^j(1-\gamma^5)\psi^j}{E_k-E_j} \tag{5}$$

At this point a standard trick, used for the Thirring model, allows us to write the four fermion interaction as the Gaussian integral of a linear one. Being neutral, each current can be bosonized separately by standard abelian bosonization [4], giving the effective Lagrangian:

$$\begin{aligned} L &= \frac{N}{8\pi}\sum_k(E_k-\frac{2}{N}\phi_k)\epsilon^{\mu\nu}(\partial_\mu A^k_\nu-\partial_\nu A^k_\mu)-\frac{N}{4\pi}g^2\sum_2^\infty f_n\sum_k E_k^n + \\ &\quad +\sum_k\frac{1}{8\pi}\partial^\mu\phi_k\partial_\mu\phi_k-mc\mu\cos\phi_k-\frac{4\pi}{N}(c\mu)^2\sum_{k>j}\frac{\sin(\phi_k-\phi_j)}{E_k-E_j} \end{aligned} \tag{6}$$

where the constants $c$ and the scale $\mu$ are regularization dependent. After continuing to the Euclidean region and adding the ghost and gluon contribution of Eq.(4), the Euclidean effective action becomes:

$$\begin{aligned} L_E &= -i\frac{N}{8\pi}\sum_k(E_k-\frac{2}{N}\phi_k)\epsilon^{\mu\nu}(\partial_\mu A^k_\nu-\partial_\nu A^k_\mu)+\frac{N}{4\pi}g^2\sum_2^\infty f_n\sum_k E_k^n + \\ &\quad -\frac{1}{4\pi}\sum_{k>j}R\log(E_k-E_j)^2 + \\ &\quad +\sum_k\frac{1}{8\pi}\partial^\mu\phi_k\partial_\mu\phi_k+mc\mu\cos\phi_k+\frac{4\pi}{N}(c\mu)^2\sum_{k>j}\frac{\sin(\phi_k-\phi_j)}{E_k-E_j} \end{aligned} \tag{7}$$

As a last step, integrating away, in an axial gauge, the neutral $U(1)$ gauge fields that enter the first order Lagrangian as Lagrange multipliers, gives the constraint:

$$E_k=\frac{2}{N}\phi_k+\theta_k \tag{8}$$

This constraint identifies the electric field with the bosonized fermion field, up to a rescaling and a shift by a $\theta$ angle which appears also in the Schwinger model [2]. With the $\theta$ set to zero, the final result for the (Euclidean) meson effective action is:

$$
\begin{aligned}
L_E &= \frac{N}{4\pi} g^2 \sum_2^\infty f_n \sum_k \left(\frac{2\phi_k}{\sqrt{N}}\right)^n - \frac{1}{4\pi} \sum_{k>j} R \log(\phi_k - \phi_j)^2 + \\
&+ \sum_k \frac{N}{8\pi} \partial^\mu \phi_k \partial_\mu \phi_k + c\mu m \cos(\sqrt{N}\phi_k) + \\
&+ \frac{2\pi}{\sqrt{N}} (c\mu)^2 \sum_{k>j} \frac{\sin(\sqrt{N}\phi_k - \sqrt{N}\phi_j)}{\phi_k - \phi_j}
\end{aligned}
\tag{9}
$$

where I have rescaled the $\phi_k$ fields by a factor of $\sqrt{N}$.
A few comments are in order.
At the leading $1/N$ order, the Lagrangian of Eq.(7) reproduces the topological Lagrangian of $YM_2$ [5], if configurations with non-trivial $U(1)$ magnetic charge are included [3]. In this case, indeed, the $E_k$ fields become integral valued in the dual of the lattice of the magnetic charges up to a constant factor.
If the integral nature of $E_k$ is ignored, in the large-$N$ limit the structure of the 't Hooft equation is reproduced, up to the regularization dependent terms, at least for the massless case. Indeed the 't Hooft equation:

$$
\begin{aligned}
p^2 \chi(x) &= M^2 \left[\frac{1}{x} + \frac{1}{1-x}\right] \chi(x) - \frac{g^2}{\pi} P \int_0^1 \frac{\chi(y)}{(x-y)^2} \\
M^2 &= m^2 - \frac{g^2}{\pi}
\end{aligned}
\tag{10}
$$

contains two terms, the kinetic one and the potential one, that can be easily traced back to the kinetic term for $\phi_k$ and the second derivative of the logarithm of the Vandermonde determinant in Eq.(9). More care and further study is necessary to trace back the term proportional to $m^2$ in the 't Hooft

equation to the regularization dependent terms in Eq.(9). They may well contribute the necessary quadratic term in the 't Hooft equation. However, since their coefficients are regularization dependent, it is necessary to find a way to fix them unambiguously. Since the regularization implicit in the bosonization need not preserve chiral symmetry, there should possibly be a mixing between them. The mixing coefficients should be determined using chiral Ward identities, in the spirit of [6]. Notice that some care about signs has been necessary for comparing the 't Hooft equation that is defined in Minkowski space, with the Lagrangian of Eq.(9) that is the Euclidean one. However, for a complete comparison of the structure and the coefficients of the bosonized effective action with the 't Hooft equation, it is necessary to compute carefully the quadratic fluctuations around the master field for this action. This necessarily involves the density of the eigenvalues and it is left for further investigation.
The integral nature of $E_k$ allows probably a stringy interpretation of $QCD_2$ analogous to $YM_2$ [7], in which this time the $QCD$ string propagates real degrees of freedom and the partition function does not count only covering maps of Riemann surfaces.
It is also clear that the condensation of the eigenvalues, that occurs at strong coupling in $YM_2$ [8], modifies the 't Hooft equation, in the strong coupling limit, via the different density of eigenvalues. I plan to consider these matters in more detail in a forthcoming paper.

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The author wishes to thank E. Abdalla and C. Abdalla for mentioning, while this paper was typewritten, ref. [9] and ref. [10], where the four fermion interaction term in the effective action was already obtained. However, the complete meson effective action in the abelian bosonized form and the link with the 't Hooft equation seem to be new in the literature. It should also be mentioned that in ref. [11] an alternative approach based on non-abelian bosonization is pursued; it would be interesting to find out a relation with the abelian bosonization.
In addition, in ref. [12], the 't Hooft equation has been obtained using functional techniques in the light-cone gauge.

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